**Course Methods test 3 Year 12**

Student name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Teacher name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Task type: Response**

**Time allowed for this task: \_\_\_40\_\_\_\_\_\_\_\_ mins**

**Number of questions: \_\_\_\_8\_\_\_\_\_\_\_**

**Materials required:** Calculator with CAS capability (to be provided by the student)

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates, notes on one unfolded sheet of   
A4 paper, and up to three calculators approved for use in the WACE examinations

**Marks available: \_\_\_46\_\_\_ marks**

**Task weighting: \_10\_\_\_%**

**Formula sheet provided: Yes**

**Note: All part questions worth more than 2 marks require working to obtain full marks.**

Q1 (3 marks)

The expected value of the discrete probability distribution given below is 2.8. Determine the values of  and hence determine Var(), the variance of .

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 |
| P() | 0.1 |  | 0.2 |  | 0.1 |

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 sets up one equation with p&q  🗸 sets up two equations with p&q  🗸 solves for both p&q  (Note: max 2 marks if no working shown) |

Q2 (9 marks)

A students wishes to play a gambling game on mufti day involving throwing two regular fair dice, each numbered 1 to 6. To play the game the student must pay $2 for each throw of two dice. If they score a double i.e two 1s, two 2s etc they win $6. If they throw a total of 7 they win $11 and anything else they receive nothing.

Let $ equal the profit a player receives on a single play.

1. Describe the random variable . (1 mark)

|  |
| --- |
| **Solution** |
| Discrete random variable |
| **Specific behaviours** |
| 🗸 states discrete |

1. Complete the following table for . (3 marks)

|  |
| --- |
| **Solution** |
| |  |  |  |  | | --- | --- | --- | --- | |  | $9 | $4 | -$2 | | P() |  |  |  | |
| **Specific behaviours** |
| 🗸 correct values for x  🗸 at least 1 probs correct  🗸 all probs correct |

1. Determine the expected profit by a player on a single game. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 multiplies x by prob  🗸 shows sum of products  🗸 states expected profit, approx. or exact (no need for units)  (Note two marks for answer only) |

1. Determine the standard deviation of . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 shows calculation  🗸 determines variance  🗸states standard deviation  Note: Answer only two marks- third mark refers to shown working |

Q3 (7 marks)

A factory produces toy cars. The probability that any toy car being defective is 0.15. If 20 toy cars are selected at random, let  equal the number of defective cars out of 20.

1. Describe the distribution . ( 2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 states Binomial  🗸 states n & p |

1. Determine that probability that exactly 4 cars will be defective. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 uses correct parameters  🗸 states prob |

1. Determine the probability that at least 4 cars will be defective given that we know at least 2 cars are defective. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 uses conditional prob reasoning  🗸 shows numerator and denominator values  🗸 states prob |

Q4 (4 marks)

Sound loudness,  dB, is measured by comparing the intensity of the sound, , with the intensity of a sound that is just detectable by the human ear, .

.

1. If the noise level in a room was 65 dB, express the intensity of sound in this room in terms of .. ( 1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 states expression |

1. How many times is the intensity of a 105 dB noise level that of the intensity of a 35 dB noise level? (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 uses index form  🗸 shows the powers of 10 for both levels  🗸 states simplified ratio |

Q5 (5 marks)

Below is a graph of  where  is a positive constant.

1. Sketch on the axes above  labelling major features. (2 marks)

|  |
| --- |
| **Solution** |
| (Not drawn to scale) |
| **Specific behaviours** |
| 🗸 labels new x intercept at x=6 AND appears to translate to the right  🗸 shows a dotted line as asymptote and labels equation |

1. Determine the values of  given that  contains points  and has a vertical asymptote at . ( 3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 states value of  🗸 sets up equations containing  🗸 states exact values of |

Q6 (6 marks)

1. Determine . (simplify) (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 shows use of product rule  🗸 at least one product correct  🗸 states simplified derivative |

1. Using your result in a) above and **NOT using your classpad** determine

. Show all working. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 uses linearity (integrates exp in (a) above)  🗸 integrates squared term and adds a constant  🗸 obtains exp for required integral (no need to factorise)  (zero marks for answer only- from classpad) |

Q7 (6 marks)

Consider the continuous random variable  and its probability density function shown below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 1. Determine the exact value of . (2 marks)  |  | | --- | | **Solution** | |  | | **Specific behaviours** | | 🗸 uses total area of one  🗸 solves for exact value of k |  1. Determine Prob   (4 marks) |

|  |
| --- |
| **Solution** |
| **Prob = 0.4601** |
| **Specific behaviours** |
| 🗸 determines equation of one side  🗸 determines equations of both sides  🗸 states integrals with correct limits for total area  🗸 states approx. area to 4 decimal places (accept exact) |

Q8 (5 marks)

Consider a continuous random variable, , that has the following probability density function.

 with  being constants.

1. Determine the cumulative distribution function, **,** in terms of .

(2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| 🗸 integrates with correct limits (no need to change variables)  🗸 states cumulative function |

1. Given that  solve for approximate values of to two decimal places.

(3 marks)

|  |
| --- |
| **Solution** |
| a=0.30 & b=0.17 |
| **Specific behaviours** |
| 🗸 sets up equation using total prob of one with x=5  🗸 sets up second equation at x=3  🗸 solves for a & b to two decimal places **(must round)** |